Workshop on the Combinatorial, Algorithmic and Probabilistic aspects of Partition Functions

27-28 March, 2025

Thursday

Viresh Patel (Queen Mary, University of London)
Improved bounds for the zeros of the chromatic polynomialic polynomial
Thusday, 10:00-10:30

We prove that for any graph G of maximum degree at most Δ, the zeros of its chromatic polynomial $\chi_G(z)$ (in $\mathbb{C}$) lie outside the disk of radius 5.93Δ centered at 0. This improves on the previously best known bound of approximately 6.91Δ.
We also obtain improved bounds for graphs of high girth. We prove that for every g there is a constant K_g such that for any graph G of maximum degree at most Δ and girth at least g, the zeros of its chromatic polynomial $\chi_G(z)$ lie outside the disk of radius $K_g\Delta$ centered at 0 where K_g is the solution to a certain optimisation problem. In particular, $K_g < 5$ when $g \geq 5$ and $K_g < 4$ when $g \geq 22$ and K_g tends to approximately 3.85 as $g \to \infty$.
Key to the proof is a classical theorem of Whitney which gives a combinatorial description for the coefficients of the chromatic polynomial. This is based on joint work with Matthew Jenssen and Guus Regts.

Kuikui Liu (MIT)
Strong spatial mixing for colorings on trees and its algorithmic applications
Thursday, 10:35-11:05

Abstract: Correlation decay is a fundamental and important property of distributions arising in statistical physics and theoretical computer science. A longstanding conjecture is that the uniform distribution over proper q-colorings on any tree of maximum degree Δ exhibits a strong form of correlation decay whenever $q \geq \Delta + 1$. It is surprising that such a basic question is still open, but then again it also highlights how much we still have to learn about random colorings. In this talk, I will discuss a near-resolution of this conjecture, as well as its algorithmic implications for sampling colorings on bounded-degree graphs via Glauber dynamics. Based on joint work with Zongchen Chen, Nitya Mani, and Ankur Moitra.

Ágnes Backhausz (ELTE and Rényi Institute)
Graph limits and partition functions
Thursday, 11:50-12:20

Abstract: Graph limit theory turned out to be a powerful tool to understand the asymptotic behavior of growing graph sequences, certain random graphs or spectral properties of graphs. In particular, by defining various distance notions for graphs, we can obtain convergent graph sequences together with their limit object, which is not necessarily a graph; it is often an analytic or probabilistic object. In addition, there are several graph parameters which are continuous with respect to certain graph limit notions, and which can be represented as partition functions. In the talk we will briefly present basic notions and recent results of graph limit theory focusing on its connections to partition functions.

Leslie Ann Goldberg (University of Oxford)
Sampling from the random-cluster model on random regular graphs
Thursday, 14:00-14:30

Abstract: This talk focusses on the performance of Glauber dynamics on random regular graphs for the random cluster model (with q¿2). (These topics will be defined in the talk - no particular background will be assumed.) It is known that there are temperatures where Glauber dynamics mixes slowly because it can get stuck at local maxima in the configuration space (for example, just above criticality, ordered configurations dominate but if you start Glauber dynamics at a disordered configuration it is likely to stay there for a long time, so it does not converge quickly to its stationary distribution). On the other hand, Helmuth, Jenssen and Perkins have give a polynomial-time algorithm for approximating the partition function for large q for all temperatures, based on appropriately focussing on the disordered or ordered phase (and using cluster expansion). Also, Gheissari and Sinclair obtained fast mixing for the Ising model ($q = 2$) at low temperature by focussing on the ordered phase. In this work, which is joint with Andreas Galanis and Paulina Smolarova, we consider, for each temperature, starting Glauber dynamics in the appropriate phase, and using arguments based on polymers, we demonstrate fast mixing (for sufficiently large q with respect to the degree).

Marcus Pappik (Hasso Plattner Institute, University of Potsdam)
Fast and Slow Mixing of the Kawasaki Dynamics on Bounded-Degree Graphs
Thursday, 14:35-15:05

Abstract: We study the worst-case mixing time of the global Kawasaki dynamics for the fixed-magnetization ferromagnetic Ising model on the class of bounded-degree graphs. Our result partially proves and disproves a conjecture of Carlson, Davies, Kolla, and Perkins: While we confirm that the Kawasaki dynamics mix rapidly below the tree uniqueness threshold for all magnetizations, we also show that contrary to their conjecture, the regime of fast mixing does not extend throughout the entire regime of tractability of the model. Our rapid mixing result involves showing that a sufficiently strong notion of zero-freeness of the partition function of the grand-canonical Ising model implies spectral independence in the fixed-magnetization model. For the slow mixing

part, we first prove a sharp threshold for the existence of multiple metastable states in the grand-canonical Ising model on random regular graphs, which we then use to conclude metastability on a suitable class of random graphs for the fixed-magnetization setting.

Tyler Helmuth (University of Durham)
Pirogov–Sinai Theory Beyond Lattices
Thursday, 15:30-16:00

Abstract: Independent sets are of interest in both statistical physics and computer science; in the former as a discrete model of crystallization, and in the latter as a constraint satisfaction problem. This common interest has lead to some fruitful interactions between the two fields, and it motivates the study of random independent sets (aka: the hard-core lattice gas) on rather general bipartite graphs. I'll explain this motivation, which lead Sarah Cannon, Will Perkins, and myself to develop Pirogov–Sinai theory beyond it's traditional setting. Using this tool we are able to discuss phase coexistence (and more) for the hard-core lattice gas in some generality.

Friday

Ewan Davies (Colorado State University)
Occupancy fractions: graph theory and algorithms
Friday, 9:30-10:00

Abstract: We review some of the applications of bounds on occupancy fractions or marginals of the hard-core model in graph theory and algorithms.

Heng Guo (University of Edinburgh)
Deterministic counting from coupling independence
Friday, 10:05-10:35

Abstract: We will introduce a new deterministic approximate counting algorithm, based a recursive LP certification method. Applications include efficient deterministic algorithms for counting colourings in regimes matching their randomised counterparts.

Noela Müller (Eindhoven University of Technology)
The number of random 2-SAT solutions is asymptotically log-normal
Friday, 11:20-11:50

Abstract: In this talk, I will present a result that demonstrates that throughout the satisfiable phase, the logarithm of the number of satisfying assignments of a random 2-SAT formula (which can be regarded as a partition function) satisfies a central limit theorem. This implies that the log of the number of satisfying assignments exhibits fluctuations of the order of the square root of the number of variables. The formula for the variance can be evaluated effectively. By contrast, for numerous other random constraint satisfaction problems the typical fluctuations of the logarithm of the number of solutions are bounded throughout all or most of the satisfiable regime.
This is joint work with Arnab Chatterjee, Amin Coja-Oghlan, Connor Riddlesden, Maurice Rolvien, Pavel Zakharov and Haodong Zhu.

Mikhail Hlushchanka (University of Amsterdam)
The independence polynomial on recursive graph sequences - the dynamical perspective
Friday, 13:30-14:00

Abstract: The distribution of the zeros of partition functions on graphs are intimately related to the analyticity of physical quantities and their phase transitions. In their pioneering work in the 1950s, Lee and Yang proved that the free energy per site of the cubic lattice is analytic at a given positive real parameter, provided that the complex zeros of the partition functions for a sequence of finite graphs converging to the lattice avoid a neighborhood of this parameter.
In the last decade, a special focus was put on graph sequences that do not converge to a regular lattice but are instead defined recursively. Examples of such recursive graph sequences include hierarchical lattices, Cayley trees, Sierpiński gasket graphs, and various self-similar Schreier graphs. Significant progress has been made in studying the partition functions of the Ising, Potts, and Hard-

Core models on these graphs (with the latter two corresponding to the chromatic and independence graph polynomials, respectively). One main advantage of working with recursive sequences of graphs is that the underlying recursion naturally induces an iterative system on the level of partition functions, often given in terms of rational maps in one or several (complex) variables. By analyzing the dynamical behavior of these iterative systems, we can gain insights into the properties of the respective graph polynomials, such as phase transitions and computational complexity.
In our current work with Han Peters (University of Amsterdam), we attempt to establish a unified framework for recursive graph sequences in the Hard-Core model setting. We start with an arbitrary graph with k marked vertices. At each recursive step, we construct a new graph by taking n copies of the previous graph and connecting these copies along the marked vertices (according to a specified rule). The dynamical systems that emerge from the respective independence polynomials are represented by homogeneous polynomials of degree n in 2^k variables. Somewhat surprisingly, it turns out that these systems can be successfully analyzed in this general context. In the talk, I will report on our results on the structure of the zero sets of the independence polynomials for these recursive graph sequences, particularly highlighting the absence of phase transitions.

Konrad Anand (University of Edinburgh)
Sink-free orientations: a local sampler with applications
Friday, 14:05-14:35

Abstract: On graphs of minimum degree at least 3, we show that there is an efficient deterministic approximate counting algorithm, a near-linear time sampling algorithm, and an efficient randomized approximate counting algorithm for sink-free orientations. All three algorithms are based on a local implementation of the sink popping method (Cohn, Pemantle, and Propp, 2002) under the partial rejection sampling framework (Guo, Jerrum, and Liu, 2019).

Márton Borbényi (ELTE & Rényi Institute)
Random cluster model on locally tree-like regular graphs: free energy and typical landscape
Friday, 15:20-15:50

Abstract: We show that if $(G_n)_n$ is a sequence of d-regular graphs such that the girth $g(G_n) \to \infty$, then the energy of the random cluster model, $\frac{1}{v(G_n)} \ln Z_{G_n}(q, w)$, converges to some value $\ln \Phi_{d,q,w}$ if $q \geq 2$ and $w \geq 0$.
In particular, we describe the free energy $\ln \Phi_{d,q,w}$, together with the phase transition and the typical landscape of the random cluster model. Our main tools are large deviation type inequalities for the Ising model, combined with a connection between the random cluster model and the Ising model. The same conclusion holds true for a sequence of random d-regular graphs with probability one.
Ongoing work with Ferenc Bencs and Péter Csikvári.